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Some Lower Bounds of the Ramsey Numbers $n(k, k)$

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Lower bounds for the Ramsey numbers $n(k, k)$, $k = 5,..., 9$, are given.

We follow the notation of [2]. Let $n(i, j)$ denote the smallest integer such that, if G is a complete graph on at least $n(i, j)$ points with the edges colored red and blue, then G either contains a complete red graph on i points or a complete blue graph on j points. Given a finite field F on n elements, one can color a complete n graph with two colors as follows: Identify the points of G with the elements of F and color an edge (u, v) of G red if $u - v$ is a quadratic residue, blue otherwise (provided -1 is a quadratic residue). Greenwood and Gleason [2] used the integers modulo 5 and 17 to establish that $n(3, 3) \geqslant 6$ and $n(4, 4) \geqslant 18$, respectively; they also showed that $n(3, 3) = 6$ and $n(4, 4) = 18$. Graver and Yackel [1] proved that the only (3, 3) or (4, 4) colorings of a complete graph on 5 or 17 points, respectively, are isomorphic to graphs determined by quadratic residues.

Using the computer facilities at State University College at Oswego, we considered the fields Z_p, p prime in the form $4r + 1$ and colored graphs by using quadratic residues, thereby obtaining lower bounds for various $n(k, k)$. The bounds for $n(5, 5)$ and $n(6, 6)$ were given in [3].

Our results are summarized in Table I. In the second row we give the largest prime p we found which is less than $n(k, k)$.

TABLE I

k	5	6	7	8	9
p	37	101	109	281	373

REFERENCES

1. J. E. GRAVER AND J. YACKEL, Some graph theoretic results associated with Ramsey's theorem, *J. Combinatorial Theory* **4** (1968), 125–175.
2. R. E. GREENWOOD AND A. M. GLEASON, Combinatorial relations and chromatic graphs, *Canad. J. Math.* **7** (1955), 1–7.
3. J. G. KALBFLEISCH, Upper bounds for some Ramsey numbers, *J. Combinatorial Theory* **2** (1967), 35–42.